An Exploratory Investigation into the Difficulties of Solving 'Guessing Game' Combination Problems

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Abstract

This study investigates the challenges faced by primary-level students in learning mathematics, specifically focusing on why an entire class struggled with basic mathematical concepts. The research aimed to explore how students approached simple "guessing game" problems and to identify the barriers to their understanding. The study involved interviews with 27 students, both individually and in pairs, along with classroom observations to detect common difficulties. From this group, four students, whose challenges were representative of the entire cohort, were selected for more in-depth interviews. Key findings revealed several common mathematical challenges: lack of flexibility in thinking, difficulty using counting strategies like counting on, inability to translate word problems into number sentences, and struggles with operating number sentences once written. Additionally, many students had trouble with grouping and regrouping numbers or prematurely using number sentences without a proper grasp of the concepts involved. In a busy classroom setting, with numerous students and a curriculum to follow, it is easy for teachers to overlook the deeper reasons behind a student's struggles with mathematics. Teachers might mistakenly assume a student simply made a calculation error, when in reality the student may not have understood the problem at all. The study highlights the importance of recognizing and addressing these fundamental issues to improve mathematical learning outcomes for students.

Keywords: *Mathematical difficulties; Primary education; Student understanding; Word problems; Classroom challenges*

1 INTRODUCTION

As a mathematics educator with over two decades of teaching experience, primarily at the primary school level in various institutions in Hong Kong, I have consistently observed a wide range of abilities among students, particularly in their struggles to comprehend fundamental mathematical concepts. Mathematics is integral to daily life, encompassing tasks such as identifying bus numbers, interpreting time, understanding quantities, and managing money. These everyday interactions underscore the importance of developing mathematical skills from an early age. Despite early exposure, some students demonstrate the ability to grasp basic concepts in one context but struggle to transfer this understanding to analogous situations (Hong Kong Examinations and Assessment Authority, 2019; Geesa et al. (2019). This inconsistency raises critical questions: Do these challenges arise from inherent cognitive limitations, ineffective instruction, or a lack of student engagement?

To investigate the underlying causes of students' difficulties in understanding and applying basic mathematical concepts flexibly, this study focused on primary school students in Hong Kong. The primary objective was to contribute insights that could inform and improve teaching practices for mathematics educators. The findings revealed that many students faced significant challenges when solving elementary arithmetic word problems, reflecting a broader issue of mathematical underachievement. Although factors such as cultural influences, language barriers, and instructional quality may play a role in these difficulties, the scope of the study was limited to examining students' behaviors and cognitive processes while engaging with fundamental mathematical tasks. By identifying patterns in their strategies and errors, the study aimed to provide evidence-based recommendations to enhance instructional approaches.

1.1 Research Question

To investigate the challenges faced by students, this study aimed to address the following research question:
What difficulties did the children in this study encounter when solving guessing game problems?

To identify these challenges, elementary mathematics problems were utilized to examine students' difficulties in the subject. Specifically, the study focused on observing how children performed, the strategies they employed, and the points at which they became confused or disoriented when attempting particular types of questions, particularly guessing game problems. For guessing game problems, the analysis centered on how students discerned the possible components of a given number. These observations served to identify the nature of the difficulties ("what" the challenges were). Subsequently, the study explored the underlying causes ("why" the difficulties occurred) in the context of each problem, such as misunderstanding the part-whole relationship, inability to subitize, or incorrect application of mathematical procedures.

2.0 LITERATURE REVIEW

Neuman's study is one of the most frequently quoted studies in Marton's works on phenomenography and variation theory (Marton & Booth, 1997; Marton & Tsui, 2004, Aksu & Kul, 2019). Neuman's experiment confirms that 'children develop without formal instruction the concepts they need to solve their everyday problems, adequately and efficiently:

… if teachers use the children's concepts as the starting point for more advanced thinking, then the teaching of mathematics will assuredly result in the learning of mathematics (Neuman,1987).

Neuman (1987) found how numbers are seen and handled by young children, in terms of conceptions and different critical aspects. Neuman attempted to discover the reason for the failure of some children to learn the four basic operations of arithmetic. Whilst looking at the origins of arithmetic skills, she started with the assumption that, before we could ascertain the origin of arithmetic skills, we must first understand how children perceive the numbers 1 to 10. Neuman was able to distinguish different ways by which children understand the numbers 1 to 10. She saw these ways as forming a system within which there is a subset of understandings of numbers 1 to 10. These understandings have in common the fact that they do not provide a framework for getting the right answer to simple arithmetic problems. She termed these understandings 'early numerical' conceptions.

The guessing game was the most powerful problem devised by Neuman (1987). The guessing game was found to be particularly illuminating as regards a child's ability to understand part-whole relations. 9 coins were separated into 2 groups unknown to the child and put into two boxes. The child was asked to guess what could be the possible combinations of numbers of the buttons within 2 boxes. Although children may guess any number of combinations, the goal is for the child to see whether the child can attend to the one hidden invariable: the whole, namely the number of coins distributed between the two boxes (e.g., 9), yet at the same time, to the parts which can vary with the certain inter- dependence relation between them, so that together they made 9. In this game, through the different guesses that the child freely makes, the researcher can find out how different numbers relate to their conception of numbers. This is how I see the beauty of the guessing game.

As regards the guessing game, there were certain differences between my study and Neuman's. Neuman's students were all aged seven and she did the guessing game with them at the beginning of the first semester. My students were aged between seven and nine and I gave them the guessing game on two occasions – the first being at the beginning of the first semester and the second being at the end of the second semester. Neuman interviewed 100 students whereas I interviewed 27. Neuman's guessing game was based on only one total: 9 – I also used 9 but, depending upon a student's ability, used different totals – sometimes lower, sometimes greater.

3.0 METHODOLOGY

The research primarily involved face-to-face interviews, typically conducted on a one-on-one basis, though occasionally in pairs. During these interviews, children were presented with guessing game problems and their problem-solving approaches were observed. No instruction or tutoring was provided during the sessions; the process was purely observational, with interactions limited to prompting, questioning, and responding (Boyce & Neale, 2006; Lindvall & Ryve, 2019). All interviews were recorded using audio and/or video for subsequent analysis.

The study aimed to gain a deeper understanding of the difficulties children encounter in mathematical problem-solving. To achieve this, think-aloud interviews were employed, allowing children to articulate their thought processes while solving mathematical word problems (Zazkis & Hazzan, 1998). This methodology provided insights into their reasoning and the challenges they faced.

3.1 Participants

The study involved students from Year 3 and Year 4, with each class consisting of approximately 20 to 25 pupils. The language of instruction was English. A total of 27 students were selected for the research, comprising 9 females and 18 males, with 16 students from Year 3 and 11 from Year 4. The selection was based on their performance in the previous year's final Mathematics examinations, with scores ranging from 28% to 82%. The students were between 7 and 9 years old. Labeling students with an 82% score as "underachievers" can stem from various perceptions and contexts within a school environment. The school has set a high academic standard, an 82% is viewed as falling short of those expectations, particularly if a significant number of students achieve higher scores. In a competitive academic environment, students scoring 82% is compared unfavorably to their peers who consistently achieve higher scores, leading to a perception of underachievement.

3.2 Instruments

The study employed a series of guessing games similar to those utilized by Neuman. For the activity, nine coins were placed on a table, and the children were asked to confirm the total number of coins present. After counting and verifying that there were indeed nine coins, the coins were removed. Subsequently, the coins were divided discreetly, with some placed into one box and the remainder into another. The exact number of coins placed in each box was not disclosed. The children were informed that the nine coins had been divided between the two boxes, and they were then asked to estimate the quantities in each box.

The children were given five attempts to determine the distribution of coins between the two boxes. This approach was chosen because it was assumed, correctly, that the children had not previously encountered this type of activity. Allowing five guesses encouraged trial-and-error learning and provided valuable insights into the strategies the children employed and the difficulties they experienced with numerical concepts, decomposition, and part-whole relationships. Depending on the student's proficiency, the game was repeated using different total quantities of coins to further explore their problem-solving approaches.

3.3 Data Analysis

Each student's performance was evaluated individually, and each problem was analyzed systematically. This evaluation involved consulting field notes, reviewing any annotations or calculations made by the children on paper, and analyzing video recordings of the sessions. The challenges encountered by the children during the first problem were documented, and their performance on the second problem was then assessed to determine whether those initial difficulties persisted or if new challenges emerged. This process was repeated for the third problem, ensuring a comprehensive analysis of the students' problem-solving approaches and the progression of their difficulties across the tasks.

We look at problems students had when they were asked to decompose a given whole number into two parts. Different numbers were used as the whole from time to time. Before posing the problem verbally, I placed the total number of coins on the table (e.g. 9 coins as given in the first question for this problem type) and asked them to count and then write down the number they found (9). I wanted the children to be clear that we were starting with nine coins. I then took the coins away. I then told them that I was putting some of those nine coins in one box and the rest in another. Both boxes had lids. I closed the lids. I then asked them to say how many coins there might be in the first box and then how many there might be in the second box. I should stress that I did not ask them to say 'how many there might therefore be in the second box after they told me the number for the first box'. I wanted to discover whether the children, themselves, would work out that there was actually a 'therefore' issue here. By this, I mean that I wanted to see if the child would, first of all, suggest a number (below nine) for box 1 and then realize that the number for the second box must, therefore, be such that, when combined with the first number, the result was 9. This, of course, presupposed that the child realized that it was a question concerning the decomposition of a whole number into the parts and also certain logical relations between the parts because of the whole.

By 'verbally', I mean problems were presented orally, including some materials and props such as coins, boxes – without the addition of pictures or the written word. For example, I would ask the students questions like: 'You have three coins. The book you want to buy costs 12 coins. How many more coins do you need'? It was during the interviews that the various word problems were put to the students. I was careful not to lead a child to the correct answer, or to lead the child into using a particular procedure or strategy. I simply questioned or probed when a student gave me their solution. If a student appeared not to understand the question I might rephrase it. If a child was finding real difficulty in coming up with a solution or, perhaps, seemed to have no idea where to start, I might give a hint such as: 'can you combine these two numbers?' Or, 'how many altogether?' Or, 'how many/much more?' In the guessing game, if a child could not even think of a number to guess for the first box, I might say: 'well, just give me any number that is possible'.

After the children had made all their suggestions, I then asked the students to check their answer after they had completed the guesses to see whether they were reasonable before I opened the boxes, because I wanted to see how many different and correct combinations students could suggest altogether. I then opened the boxes and got the students to count how many were in each box. This was an exercise in assessing their ability to decompose a whole number into two parts in different combinations.For the four students selected for in-depth analysis (Pakiza, Addina, Barrian, and Zaima), their performance was evaluated relative to one another across the three challenges. Additionally, the difficulties exhibited by these four students were compared to those observed among the remaining 23 participants to determine whether similar challenges were shared across the group.

Following a detailed analysis of the interview data, the performance of these four students was meticulously compared with that of the remaining participants. This comparison aimed to identify commonalities and specific discrepancies, as well as to uncover any additional challenges students faced in understanding and solving the problems.

The participants attended a primary school where English served as the medium of instruction. Data for the study were collected through field observations, preparatory meetings, detailed field notes, and both audio and video-recorded interviews. Consistent with Patton's (1990) emphasis, the credibility of the study was enhanced through systematic data collection methods, triangulation, and the use of interviews, which provided high-quality qualitative data.

4.0 FINDINGS AND DISCUSSION

The interviews introduced the students to the guessing game problem. The questions were expressed verbally and with concrete manipulatives. The strategies which the students adopted in tackling the problem were observed and reported here. The results of their learning and discoveries will also be noted.

I set out the research questions as follows:

How do students discern the parts in a given number? What difficulties did the children in this study encounter when solving guessing game problems? I seek to answer this where I examine in detail the difficulties the children encountered in guessing games.

Lack of knowledge of number facts also featured in this type of problem. The failure to understand part-whole was particularly evident. Most students did not understand the idea of a unique answer and there was much evidence of a failure to understand the use of (and consequences of) variation in combinations of addends. Moreover, there is a distinct lack of the use of commutation.

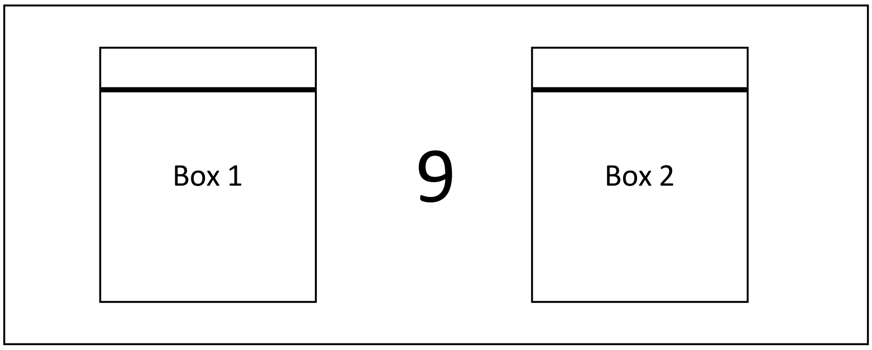

Figure 1

The guessing game was conducted using one or more values of the total (there could be 6, 9, 12, or 15 coins in total). Not every child was tested with all four numbers. Some children were tested with only one hidden total, others two, some three, and some four. In each case, they were told that they had five guesses (although, in the event, some made fewer, and others more than the five guesses per total). Those interviewed only once had only one hidden number, namely 9.

When the total number of coins was 6, some children were not able to make the full five guesses as required, because, I thought the total number was so small. Conversely, where the number was larger (e.g. 12) some students made more than five guesses although, in a few cases, even when the number was 12, a student could think of only fewer than five guesses. In such circumstances, my inference was that the children were unable to see more possibilities after the first few guesses or their knowledge of number facts was limited.

We look at the various strategies the four students (Addina, Pakiza, Barrian, and Zaima) employed when attempting the guessing game problem. We look at whether they:

- understood the commutative property
- understood the principle of unique answer
- had number of facts
- used assisting ways

- just guessed
- and how they adjusted their guesses

Understanding Commutative Property

Of the four students, only Pakiza used the commutative property. In both interviews, (five guesses in the first interview and 15 in the second) she got all her answers right in the first 15 guesses. She got three right out of five in her last five guesses. In her first ten guesses (9 coins) she commuted 4 and 5, and 3 and 6. In her next five guesses (6 coins), she commuted 5 and 1. She did not use commutative property in her last five guesses. (12 coins).

After the practice games both Zaima and Barrian (when working together) used commutation. Zaima did so on only one occasion but Barrian did so on many occasions because, by this time, she had come to understand how to solve the problem by starting with the whole and subtracting a series of different numbers. She seemed perfectly capable of ‘flipping’ any combination.

Unique Answer

Pakiza seemed to understand the concept of a unique answer from the outset. Neither Addina nor Zaima seemed to have grasped the concept of a unique answer. Barrian did not seem to understand it at the beginning, but later, when she started using subtraction, starting with the whole number, seems to have come to understand the concept of a unique answer.

Number Facts

None of the four students seemed to use number facts for the majority of their guesses. They needed to check by counting. On the rare occasions when they did use number facts, this tended to be in 'half/half' situations such as 3+3, or 4+4, or where it was just a question of increasing a number by 1 (e.g. 8+1 = 9 or, 11+1 = 12).

Assisting Ways

As indicated before, Zaima used number sentences. Neither Addina nor Pakiza used assisting ways other than using their fingers for counting. Addina did use a diagram at the beginning but soon dropped it even though it had helped her. Barry used additional coins – but only for counting purpose.

Guessing

It is like the guessing game that there would be guesses. Addina used guessing throughout. Zaima used both guessing and number sentences. Barrian departed from guesses once she had come to realize that all she had to do was start with the whole number (whatever the total number of coins there were divided between the two boxes) and then subtract one different number after another to produce the two addends to be placed in Box 1 or Box 2. Pakiza did not appear to guess at all. She started with the whole by raising nine fingers and then working out different combinations. She seemed to grasp from the outset that, whatever numbers she put in either box, the total always had to be 9.

Systematic Variation

The strategy of systematic variation used by certain students was a strategy that I had not seen previously. It seemed that those students had hit upon a new strategy – adding this to their armory. By following this strategy systematically, I suggest it led them to a better understanding of the concept of part-whole because, as I have indicated elsewhere, they had to keep in focus the fact that the result always had to be nine – and then it was a simple question of revising the numbers in each combination of addends, up or down, as appropriate. This, I concluded, showed a new understanding that the whole number could be divided into varying subsets.

Observations of all participants on the problems related to the guessing game and the strategies employed—first by the four students (Addina, Pakiza, Barrian, and Zaima) and then by the remaining 23 students—focused on identifying commonalities, specific disparities, and additional challenges they encountered, as detailed below:

Practising with 6 Coins

Many students found it easier to solve the guessing game problem when the number of coins was reduced to six. Several factors may explain this.

Firstly, subitization—the ability to instantly recognize the number of items in a small group—was likely easier with six coins compared to nine. Most students are familiar with the concept of five fingers on a hand, and adding one more is an intuitive task. Furthermore, six items, whether fingers or coins, are easier to visualize and process mentally. This scenario aligns with students' prior experiences and provides a more concrete context, making it more manageable for them to conceptualize.

When six coins were hidden, students appeared to focus carefully on the two boxes, attempting to mentally distribute the coins between them. With a smaller number, it was easier to imagine X coins in one box and simultaneously consider the corresponding Y coins in the other. This smaller total facilitated their ability to think through the problem, allowing them to attend to both parts of the equation more effectively. The students had a clearer, more direct sense of the entire situation.

Conversely, larger numbers proved significantly more challenging for the students to visualize and manage using their problem-solving strategies. As a result, many students abandoned analytical thinking and resorted to guessing. In some cases, these guesses were informed and thoughtful, while in others, they were entirely random due to the difficulty of the task.

Interestingly, working with six coins helped some students develop greater competence in solving problems involving larger numbers. This observation underscores the instructional value of real-world problem-solving activities that engage students with small, manageable quantities. Such tasks encourage meaningful thinking and build foundational skills that enable students to tackle more complex problems over time.

Differences in Advancement after Practising with Six Coins

Although most students found the guessing game easier when the total number of coins was reduced to six, some appeared to learn from this experience more effectively than others. For some students, the simpler task facilitated deeper conceptual understanding, while for others, the benefits were less pronounced. One notable example was Barrian, for whom practicing with six coins led to a significant breakthrough. After engaging with the smaller total, she was able to tackle larger whole numbers—such as 9, 12, and even 15 coins—with considerable success. This transformation highlighted her acquisition of the part-whole concept, which she applied systematically to solve problems with larger totals. Barrian’s improvement demonstrated a significant enhancement in both conceptual and procedural knowledge, with each reinforcing the other. She not only understood the need to find combinations that added up to the total but also developed a methodical approach for doing so efficiently and accurately. A key factor in mastering the guessing game was a clear understanding of the part-whole relationship. Students who recognized the underlying principle could generalize and apply it to any given total. However, not all students made this connection. Some, who performed well with six coins, struggled to transfer their understanding to larger totals, such as 9 or 12. Their performance often deteriorated as the numbers increased, suggesting gaps in their conceptual grasp.

Barrian's problem-solving process provides valuable insights. For instance, she began by guessing "3" for Box 1 and "4" for Box 2. Upon checking with her fingers, she realized the combination was incorrect. She then kept "3" fixed in Box 1 and systematically varied the number in Box 2 until the combination was correct. Afterward, she reversed the process, varying the number in Box 1 while determining the corresponding number in Box 2. This systematic approach reflects her ability to structure variation and invariance, a critical skill for solving such problems.

Another important aspect of Barrian's problem-solving strategy was her use of verification to build understanding. This characteristic was also evident in her solutions to Missing Addend Problems. In these tasks, she initially worked from the part to the whole—guessing the missing addend and counting on her fingers to check if it matched the intended total. Once she arrived at the correct total (e.g., seven fingers raised), she was able to explain her solution from the whole to the parts, separating the newly added addend from the original amount. While the part-whole concept did not guide her problem-solving from the outset, it emerged during the verification process, reinforcing her understanding.

Barrian's ability to use checking as a learning tool and her systematic exploration of possibilities exemplify how some students effectively engage with and learn from problem-solving tasks. Her experience underscores the importance of fostering strategies that allow students to connect concrete actions with abstract principles, enabling them to apply their learning to increasingly complex scenarios.

6 CONCLUSION AND IMPLICATION

In my research, I was particularly interested in investigating whether students understood that a whole can consist of different parts. This is why the guessing game was so valuable, as it provided insight into their grasp of the part-whole concept. For example, if a child can only suggest one combination (e.g., 4 + 5) for a total of nine coins, it indicates that their understanding has not yet developed sufficiently to make the necessary connections between parts of the whole. The ability to identify different combinations enhances this web of knowledge. A child who cannot find multiple combinations to make 9 demonstrates a limitation in their conceptual understanding.

I viewed the guessing game as a particularly useful tool, not only for identifying the part-whole conceptual skills children already possessed when they started school but also for helping them acquire these essential skills. The understanding of part-whole relationships is one of the most fundamental concepts children need to develop a deeper understanding of mathematics. In one instance, a child struggled while trying to find the missing addend. She made an initial guess for the missing addend, but upon checking, she found that the total was 8 instead of the target 9. She made another seemingly random guess for the missing addend but did not recognize that increasing her guess by 1 would bring the total to the desired amount. She failed to notice that the difference was just 1 and that increasing the part (the missing addend) by 1 would increase the whole by 1, thus matching the target total. This illustrates the difficulty some students face in recognizing simple numerical relationships and the importance of guiding them toward a deeper conceptual understanding.

In my research I sought to discover the difficulties encountered by primary school students with low ability in mathematics. I have made certain interpretations concerning the various procedures and strategies employed by the students in my study and how they make sense of the problem situation. I would submit that a fruitful area for further research involves understanding the way various procedures interact with each other and how they can promote an understanding of the part-whole relationship. Teachers might wish to observe how students work and check their answers to understand their thinking. They might find that relational understanding of part-whole can be built using values within 10 or 20, and with grouping and regrouping, rather than simply counting. Teachers might wish to encourage students to use multiple methods when checking their answers. When teachers are trained in the study of instructional designs, their flexible use of the materials or methods is facilitated and stimulated (Wittmann, 2021). In this way, teachers might be able to determine whether the child understands the concept rather than blindly following a particular rule.

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